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Computer Simulation of the Electron Mixing Mechanism in Ion Propulsion

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A one-dimensional simulation with electrons and ions treated as sheet charges in mutual coulomb interaction and simplified emission conditions (ions are injected with uniform velocity through a plane accelerator grid, electrons with a thermal distribution from a plane decelerator grid at ship potential) was programmed into a digital computer. The dynamic buildup of the plasma is observed; the thrust seems to be maintained almost steadily, and good spontaneous neutralization seems to take place. Nonstatic space charge fields oscillating at the electron plasma frequency seem to provide the entropy increase needed for proper mixing.

I. Introduction

THE subject of neutralization has been approached with a wide range of attitudes that have resulted in varying degrees of optimism and pessimism.

Instinctively, one feels that the heavy ions will always pull along the requisite number of electrons, provided they are made available from some source placed in or near the beam. Any unbalance of charge will result in electrostatic fields that redistribute electrons in such a way as to restore neutrality. Indeed, it would appear that this mechanism is not dependent on collisions for its operation, and the scarcity of collisions cannot be invoked against one's "instinctive" confidence in successful neutralization.

However, careful and rigorous analytical studies have been made of the problem of collision-free electron-ion mixing. They have, on the whole, yielded negative results.[1] Broadly speaking, it is found that the electrostatic field mechanism for the adjustment of electron densities results in "overshooting" by the electrons and, at best, only neutrality in-the-mean has been achieved in rigorous theoretical models of the mixing problem.[2] Whenever the electrons are released with velocities more than twice the ion velocities (and in ion engines one would want to release them with possibly three times the ion velocity), theory has failed altogether to provide a rigorous solution, i.e., a selfconsistent field distribution in the beam.

This is one cause of possible pessimism regarding efficient mixing, and extensive experimental research was devoted to checking the theoretical predictions. On the whole, these experiments have resulted in reversion to optimism and even complete disregard of the problem. The experiments, on the other hand, encountered the criticism that space conditions cannot be simulated properly in the laboratory, and distant boundaries seemed to play a major role in the theories.

In the meantime, further theoretical developments have not been able to settle with certainty the argument whether and how neutralization may occur. The search has continued for a static potential distribution in the neutralizer region, taking into account more complicated effects. The original calculations had been concerned with monoenergetic electrons and ions and had, at best, resulted in spatially varying po-

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tentials, i.e., permanently unneutralized positive and negative regions, in a one-dimensional model, under the restricted velocity conditions mentioned. The injection of Maxwellian velocity distributions of electrons into an ion stream, which itself is not completely monoenergetic, was therefore studied.

Kino[3] and Mirels[4] recently proposed a very plausible picture of the formation of a good plasma, upstream and downstream of the electron source, rather along the lines of Langmuir's original analysis of electron and ion distribution in a plasma and its sheaths. The Kino-Mirels model would explain the experimental observations on plasma formation in the neutralizer by Sellen.[5, 6] The following characteristics of such a plasma may be anticipated:

The temperature is that of the emitter, say 2300°, equivalent to $kT = 0.2$ ev, corresponding to a mean one-way electron velocity of 150 km/sec.

The potential is very close (within 0.2 v) to that of the electron emitter.

The ions and the whole plasma drift with a velocity corresponding to the potential difference between ion emitter and plasma. Assuming 3000-v Cs ions, this is 65 km/sec.

The density is that of the ion beam at this velocity; assuming 15-ma/cm^2 ion current, this means 1.4×10^{10} ions, and the same number of electrons per cubic centimeter.

The electron plasma frequency corresponding to this density is 1000 Mc/sec.

To produce the assumed Cs ion beam, one would use an accelerator grid at -5000 v, one-third of a centimeter from the ion emitter.

However, in this plasma model one has to postulate an ergodic type of behavior of the electrons everywhere, and Kino in his theory also postulates a static floating collector, as in the laboratory experiments, rather than a diffuse or moving boundary, as in space.

Derfler[11] sometime ago developed a complete Boltzmann analysis of the problem stated. It showed that in general it was not possible to meet in space the assumptions made by Kino. As in the case of monoenergetic particles, it was possible, at best, to obtain spatially periodic potentials with neutrality in-the-mean provided the mean one-way thermal electron velocity does not exceed that of the ions.

Derfler suggested, and this has been the view of many other theoreticians,[7] that neutralization, if it occurs in space, cannot be a strictly static process. Somewhere, there must be a source of entropy, i.e., a mechanism that causes ergodic behavior and increases disorder. An instability, in particular, the instability due to a cold ion stream passing through a thermal distribution of electrons, may serve to cause the necessary entropy increase as described in Ref. 8, and thus perhaps provide a well mixed beam in thermodynamic equilibrium.

In order to test this view and the hypothesis that a plasma is formed as described, the nonstatic electron-ion mixing mechanism has been programmed into an IBM 7090 computer, i.e., a computer analog of what might be the space conditions has been built. The model programmed into the

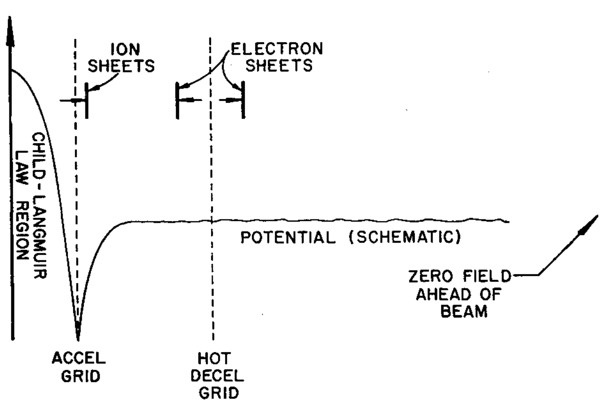

Fig. 1 One-dimensional model.

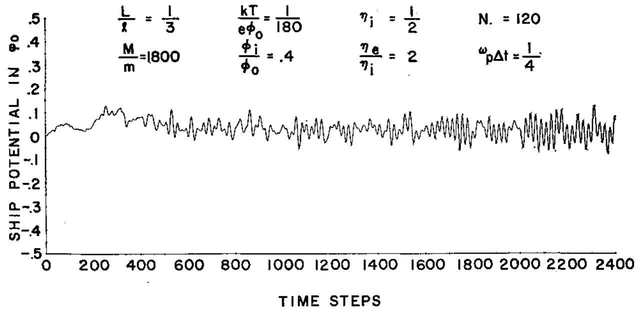

Fig. 2 Ship potential vs time, N = 120.

computer is that previously used for plasmas by Buneman[9] and Dawson.[10] The results obtained for this model led to the arguments given in Ref. 8. Similar computer experiments were reported by Dunn.[12]

A departure from realism necessitated by considerations of computer economy is the limitation of these studies to hydrogen rather than Cs beams. In order to achieve with hydrogen the same plasma drift and densities as those quoted in the introduction, one has to accelerate to 36 v across a gap of 0.029 cm, then decelerate to 21.6 v at the electron emitter.

II. Computer Model

This model is, as yet, one-dimensional: ions and electrons are idealized into sheets of charge. Cold ions are injected through an accelerator grid and electrons from a hot decelerator grid with a gaussian velocity distribution. The situation is studied both upstream and downstream of the decelerator grid (see Fig. 1).

All of the dynamical equations of the system are programmed into a high-speed digital computer with the variables in completely normalized form. These variables are as follows:

L/l = the ratio of distance between the ion emitter and the accelerator grid to the distance between the accelerator grid and decelerator-emitter grid

M/m = ratio of ion mass to electron mass

φ_1/φ_0 = the ratio of the potential between the accelerator grid and the decelerator-emitter grid to the potential between the ion emitter and the accelerator grid

$kT/e\varphi_0$ = ratio of mean electron energy to potential difference between ion emitter and accelerator grid

η_i = number of ion sheets injected per time step

η_e/η_i = ratio of electron sheets injected per time step to the number of ion sheets injected per time step

N = the approximate number of ion sheets between the accelerator grid and the decelerator-emitter grid

The output variables from the computer are as follows:

$\varphi_{\text{ship}}/\varphi_0$ = the ratio of the integrated spaceship potential at each time step to φ_0

$T_{\text{actual}}/T_{\text{ideal}}$ = the ratio of the force exerted on the ship by all the charged sheets to the ideal force on the ship calculated at each time step from the change in momentum one would expect if the ions left the ship with energy $\varphi_0 - \varphi_1$

φ_p/φ_0 = the ratio of φ_p the potential in the plasma to φ_0 as a function of distance from the accelerator grid; the potential is found by integrating the electric field in the plasma from the farthest out sheet to the accelerator grid

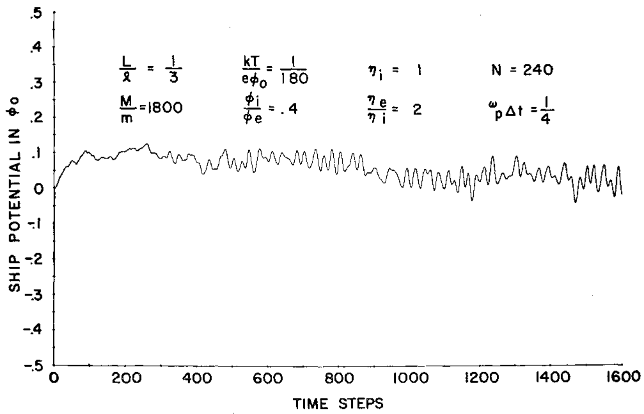

Fig. 3 Ship potential vs time, $N = 240$.

The actual numbers used in the calculations presented here for hydrogen ions are as follows:

$$L/l = \tfrac{1}{3} \qquad M/n = 1800$$

$$kT/e\varphi_0 = \tfrac{1}{180} \qquad \varphi_1/\varphi_0 = 0.4 \qquad \eta_e/\eta_i = 2$$

η_i and η_e are proportional to N. Calculations of $N = 30$, 60, 120, 240 were made.

The ion current density is half the maximum available temperature limited electron current density supplied by the hot grid. This amounts to injection of two electron charge sheets from the hot grid (alternately one upstream, one downstream) to every one ion charge sheet.

The electron sheets injected upstream are returned by the negative decelerator potential and made available for downstream mixing after passing through their emitting grid again.

However, a grid electron sheet interception rate of one in four passes is programmed for electron sheets. No ion-electron collisions have been programmed.

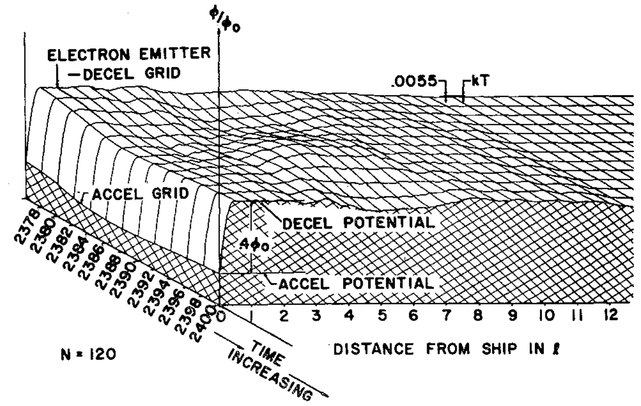

Fig. 4 Integrated potentials, time-distance plots, $N = 120$.

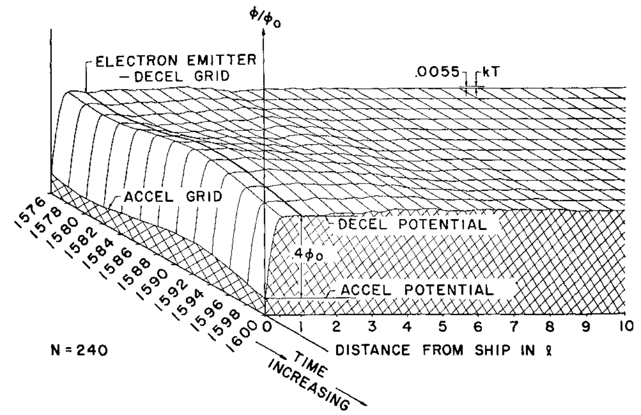

Fig. 5 Integrated potentials, time-distance plots, $N = 240$.

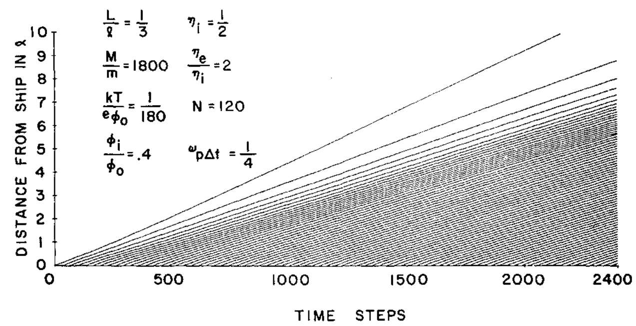

Fig. 6 Time-distance displacement graph for ions, $N = 120$.

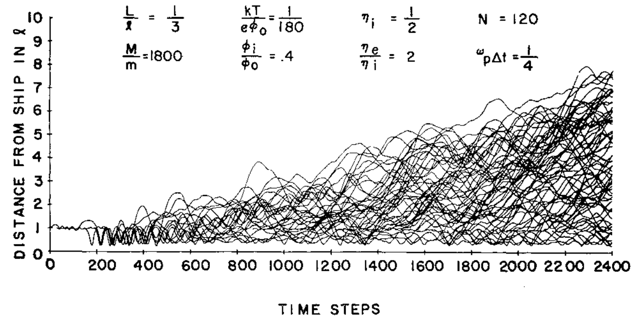

Fig. 7 Time-distance displacement graph for electrons, $N = 120$.

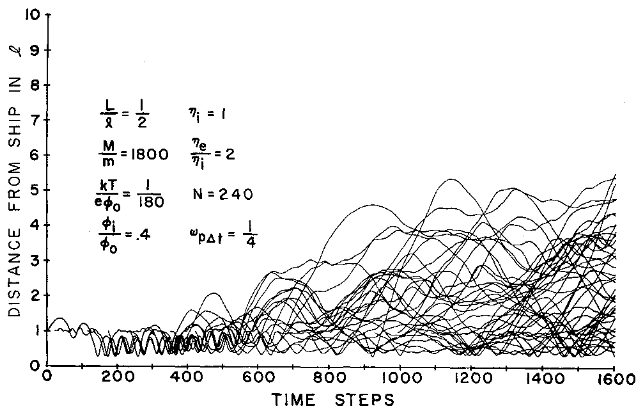

Fig. 8 Time-distance displacement graph for electrons, $N = 240$.

Injection of electrons from the hot grid began simultaneously with the entry of the first ions through the accelerator grid. The system was followed until the head of the beam had penetrated some 10 intergrid distances out into space. The debye length in the anticipated plasma is about 4% of the intergrid distance, and the one-dimensional model is sound when the grid mesh is finer than this debye length. The thrust was monitored throughout, as the actual force on the supposedly rigid grid system (see Fig. 1).

Plots Made from the Calculations

Zero field was imposed as the condition in free space ahead of the beam and the ship was left to find its own potential relative to space. This potential was monitored, (Figs. 2 and 3), although fields only were required (not potentials) for tracing the charges.

The instantaneous potential was calculated as a function of distance for many successive time steps in several cases, (Figs. 4 and 5). Time displacement graphs of some of the particles were traced automatically for several cases (Figs. 6–8).

The numerical procedure employed approximately 22 time steps per complete plasma oscillation. The charge continuum is coarse-grained into discrete sheets such that there are up to ten sheets of each species per debye length (for $N = 240$), but coarser models, down to only just over one sheet per debye length ($N = 30$), were also tried.

Electrons and ions were emitted at regular time intervals. The injection velocity of the ions was fixed, that of the electrons was picked randomly from a gaussian distribution at the specified temperature.

Results

A good plasma was formed in all cases, mixing was perfect, and that thrust maintained its ideal value after a brief initial adjustment. No electrons penetrated through the accelerator grid. They were repelled at a certain distance upstream of their source, with only ions beyond. The upstream boundary of the plasma lies exactly where predicted from the theory of self-consistent ion flow between the accelerator grid and the plasma.

No ions made turn-arounds. Electrons randomly drifted about through the entire plasma. The advancing head of the beam acts as an electron reflector. The head seems to become more and more diffuse: there appears to be a forward acceleration in space of the first ions, perhaps due to plasma pressure.

No distinct "plasma puffs" developed, i.e., self-contained advanced groups of ions holding in their own electrons. A slight tendency toward electron sheath formation at the hot grid is apparent. This sheath was intuitively predicted as a device for keeping back unwanted excess electrons. Fields (potential gradients) remained moderate throughout the plasma, and the electrons did not acquire unduly high energies above their initial kT.

However, there are marked potential fluctuations. In space these fluctuations seemed to become less violent as the number of sheets per debye length was increased. Also a longer time was needed for the fluctuations to build up when more sheets were used. In time, however, they remained at an unexpectedly high level (see Fig. 3) and occurred with electron plasma frequency.

The fluctuation amplitude in the plasma (or of the ship) amounts to one-fifth of the ion energy. Translated to the Cesium case this would mean nearly kilovolt fluctuations in the ship potential relative to space, at plasma frequency.

The potential oscillations are, in a certain sense, not unexpected. If one studies analytically the problem of a cold ion impinging on a hot plasma, one finds an instability. (The ion beam created in acceleration from even a 2000° electrode is, in fact, extremely cold in the sense of width of velocity distribution). This type of two-stream instability has often[5] been suspected as the cause of eventual turbulence and irreversible randomness in plasmas taking the place of collisions in providing irreversible, resistive, diffusive effects and heating in plasmas.

Certainly it is felt that an ion engine similar to the model described here will work in space and provide thrust. The electron plasma frequency oscillations reported here have not been confirmed by all experiments, but one must keep in mind their longitudinal and very high frequency nature and hence the difficulty in observing them. With rapidly time varying electric fields it is not correct to talk about potentials as energy functions. Even though relatively large integrated potentials appear at the ship, electron velocities much greater than the electron injection velocity are not observed. Thus, these fluctuations in spaceship potential are not detrimental to the performance of the engine but, on the contrary, provide the mechanism by which the beam is neutralized.

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